THE SLOPES

The Tour de France is the third largest sporting event in the world after the Olympic Games and the Football World Cup. Where does the attraction to this event and the popularity of cycling? Just ride on a bicycle uphill to understand it!

Definitions

What is the difference in height? This is the difference in vertical elevation between two points.

What is the gradient? This is the ratio of the difference in height of a road to its length.

Expression of the slope

The gradient can be expressed either as a percentage or a fraction or a decimal number generally between 0 and 0.25 for paved roads(*).

Example 1: a 10 % gradient (1/10 or 0.1) means that the difference in height is 10 meters for 100 meters traveled.

Example 2: the road from Malaucène to the top of Mont Ventoux (France, PACA region) has a length of 21 km and a difference in height of 1,530 meters; its average gradient is thus 7.3 % (73/1000 or 0.073).

Warning ! Do not confuse this percentage with the angle formed by the road respect to the horizontal: a 10 % gradient does not mean that this angle is 10 degrees!

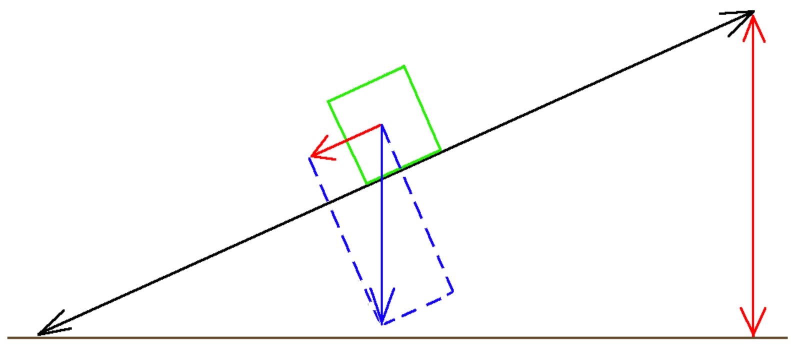
From the mathematical point of view, the slope is the sine of the angle formed by the road respect to the horizontal, to distinguish the slope at geometric sense, which is the tangent of the angle, even if the difference between these two values is negligible on the road network.

The component parallel to the road weight

What is the effect of the gradient on the movement of land vehicles?

Due to the slope, the weight has a component parallel to the road whose intensity is proportional to the sine of the angle formed by the road relative to the horizontal.

This force creates a movement downhill, it opposes the movement uphill.



Component of the weight on a slope (red arrow).

Slope and traction

The drive force is defined as the force exerted on contact with the ground by the engine to create or maintain the movement of land vehicles.

The gradient gives immediate information about the drive force to maintain the uphill movement.

Indeed, if we neglect the action of both resistive forces (rolling resistance, air resistance), the drive force must be strictly equal and opposite to the component of the weight parallel to the road, if we want to maintain a constant speed.

Example 1: a 100 kg mass cyclist (bike included) uphill on a road at 10 % gradient will apply a drive force of about 100 N, or 10 % of its weight (‘**g**’ ~ 10 $m.s^{-2}$).

Example 2: a 1,500 kilograms mass car uphill on a road at 10 % gradient will apply a drive force of about 1,500 N, or 10 % of its weight (‘**g**’ ~ 10 $m.s^{-2}$).

Slope and engine brake force

A similar reasoning applies for the purpose of stabilizing the speed in descent, if one neglects the action of resistive forces.

Example: a 1,500 kilograms mass car downhill on a road at 10 % gradient will apply an engine brake force (see ADILCA file 'engine torque') of about 1,500 N, or 10 % of its weight (‘g’ ~ 10 $m.s^{-2}$).

If the driver must reduce the speed or stop the car, the component of the weight parallel to the road is added to the braking force uphill, it is subtracted from the braking force downhill.

Example: a car that can apply a braking force of 15,000 N on a horizontal road (see the ADILCA file 'braking force') can apply, all other conditions being equal, a total braking force of approx. 16,500 N uphill on a road at 10 % gradient, about 13,500 N downhill on a road at 10 % gradient ('**g**' ~ 10 $m.s^{-2}$).

Slope and work

The work in the physical sense of the term, is defined as the energy required for moving a force.

Neglecting air resistance and rolling resistance, assuming that the transmission efficiency of the bicycle is total, the work of a cyclist on a constant slope depends only on the distance traveled.

Example 1: if we consider a cyclist mass 100 kg (bicycle included) who travels along 10 km uphill on a road at 10 % gradient, the work is about 1 MJ and therefore mobilizes equivalent muscular energy.

Example 2: if we consider a car mass 1,500 kg which travels along 10 km uphill on a road at 10 % gradient, the work is about 15 MJ and therefore mobilizes equivalent motive energy.

Slope and power

The power is defined as the work done per unit of time.

Neglecting air resistance and rolling resistance, assuming that the transmission efficiency of the bicycle is total, the power only depends on the speed with which the work is accomplished.

Example 1: if we consider a cyclist mass 100 kg (bicycle included) who travels uphill on a road at 10 % gradient at a constant speed of 4 $m.s^{-1}$ (9 mph), the power is about 400 W, a value well beyond the physical capacity of a normal individual.

Indeed, although some very talented and well-trained cyclists are able to temporarily deliver a power of 450 W, it is assumed that an effort of endurance achieved by a normal individual should not mobilize a power greater than 100 W.

Example 2: if we consider a car mass 1,500 kg which travels uphill on a road at 10 % gradient at a constant speed of 20 $m.s^{-1}$ (45 mph), the engine power required for the slope is about 30 kW (41 hp).

In reality, much greater power is required due to the rolling resistance and air resistance, the latter being proportional to the square of the velocity (see ADILCA files).

Altitude and Weather

Weather conditions (atmospheric pressure, temperature) vary with altitude. This is because the air density decreases gradually as one moves away from the ground. And when the density of air decreases, the temperature drops.

Considering the characteristics of a vertical column of air, the air density at 1,000 meters height is 12 % smaller than at the sea level.

At 2,000 meters height, the air density is 21 % smaller than at the sea level; at 3,000 meters height, it is 29 % smaller than at the sea level.

If the temperature of the air column is 20 °C (70 °F) at the sea level, it is 15 °C (60 °F) at 1,000 meters height, 10 °C (50 °F) at 2,000 meters height, 5 °C (40 °F) at 3,000 meters height, etc.

For cyclists that evolve in the mountains, these factors must be taken into consideration. The most immediate effect is the feeling of suffocation, as lungs fill is more difficult.

In addition, in equal volumes, the mass of absorbed air with each breath is reduced, that decreases muscle performance accordingly, especially as the body must also compensate for lower ambient temperature.

Therefore, except for acclimatization to increase the number of red blood cells responsible for precious oxygenation, physical efforts in the mountains are always very demanding and we must be careful not to compare sport performances in altitude with those completed at the sea level.

Of course, these physical realities have the same consequences on the operation of automobile engines since they need air to burn fuel (see ADILCA files).

(*)*The Alto de Angliru, located in the Spanish province of Asturias, is considered as one of the steepest mountain road in the world with 23 % gradient.*

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SOME RELATIONSHIPS BETWEEN QUANTITIES

Weight

$$P = M \cdot g$$

P: weight, expressed in **N**
M: mass, expressed in **kg**
g: gravitational acceleration, expressed in $m.s^{-2}$
(Earth : **g** = 9.8 $m.s^{-2}$)
consistency of the units: **P** = kg . $m.s^{-2}$ = **N**

Example: calculate the weight of a 1,500 kg mass car:

P = 1,500 x 9.8 = 14,700 N

Gradient

$$\alpha = H / L$$

α: gradient, dimensionless ;
H: difference in height, expressed in **m**
L: length of the road, expressed in **m**
consistency of the units: **α** = m^{+1} . m^{-1} = dimensionless.

Example: calculate the gradient of a 10 kilometers (6.2 miles) road with a 1,000 meters (3,000 feet) difference in height:

α = 1,000 / 10,000 = 0.1 = 1/10 = 10 %

Component of the weight

$$F = M \cdot g \cdot \alpha$$

F: component of the weight, expressed in **N**
M: mass, expressed in **kg**
g: gravitational acceleration, exprimée en $m.s^{-2}$
α: gradient, dimensionless ;
consistency of the units: **F** = kg . $m.s^{-2}$ = **N**

Example: calculate the component of the weight of a 1,500 kg (3,300 lb) mass car on a 0.1 gradient slope (**g** = 9.8 $m.s^{-2}$):

F = 1,500 x 9.8 x 0.1 = 1,470 N

Traction work

$$E = F \cdot D$$

E: work, expressed in **J**
F: component of the weight, expressed in **N**
D: distance, expressed in **m**
consistency of the units: $\mathbf{E} = N \cdot m = kg.m^{+1}.s^{-2} \cdot m^{+1} = kg.m^{2}.s^{-2} = \mathbf{J}$

Example: calculate the traction work of a 1,470 N drive force which travels upon 10 kilometers (6.2 miles):

E = 1,470 x 10,000 = 14,700,000 J

Gravitational energy

$$E = M \cdot g \cdot H$$

E: energy, expressed in **J**
M: mass, expressed in **kg**
g: gravitational acceleration, expressed in $\mathbf{m.s^{-2}}$
H: difference in height, expressed in **m**
consistency of the units: $\mathbf{E} = kg \cdot m^{+1}.s^{-2} \cdot m^{+1} = kg.m^{2}.s^{-2} = \mathbf{J}$

Example 1: calculate the gravitational energy of a 1,500 kg (3,300 lb) mass with a 1,000 meters difference in height ($g = 9.8\ m.s^{-2}$):

E = 1,500 x 9.8 x 1,000 = 14,700,000 J

Example 2: calculate the gravitational energy of a 1,500 kg (3,300 lb) mass falling free from 1,000 meters height ($g = 9.8\ m.s^{-2}$):

E = 1,500 x 9.8 x 1,000 = 14,700,000 J

Power absorbed by slope

$$B = F \cdot V$$

B: power, expressed in **W**
F: component of the weight, expressed in **N**
V: speed, expressed in $\mathbf{m.s^{-1}}$
consistency of the units: $\mathbf{B} = kg.m^{+1}.s^{-2} \cdot m^{+1}.s^{-1} = kg.m^{2}.s^{-3} = \mathbf{W}$

$$B = M \cdot g \cdot H / T$$

B: power, expressed in **W**
M: mass, expressed in **kg**
g: gravitational acceleration, expressed in **$m.s^{-2}$**
H: difference in height, expressed in **m**
T: duration, expressed in **s**
consistency of the units: **B** = $kg \cdot m^{+1}.s^{-2} \cdot m^{+1} \cdot s^{-1} = kg.m^{2}.s^{-3}$ = **W**

Example 1: calculate the power absorbed by the slope, the component of the weight being a 1,470 N force at a speed of 20 $m.s^{-1}$ (45 mph):

B = 1,470 x 20 = 29,400 W

Example 2: calculate the power absorbed by the ascension of a 1,500 kg mass upon a 1,000 m height in 10 minutes:

B = 1,500 x 9.8 x 1,000 / 600 = 24,500 W

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