THE SLOPES

The Tour de France is the third largest sporting event in the world after the Olympic Games and the Football World Cup. Where does the attraction to this event and the popularity of cycling? Just climb a bicycle by side to understand it!

Definitions

What is the difference in height? This is the difference in elevation between two points.

What is the slope? This is the ratio between the gradient of a road and its length.

Expression of the slope

The slope is a ratio that can be expressed either as a percentage or a fraction or a decimal number generally between 0 and 0.25 for paved roads(*).

Example 1: a 10 % gradient (1/10 or 0.1) means that the change in altitude is 10 meters to 100 meters traveled.

Example 2: the road Malaucène to the top of Mont Ventoux (PACA region) has a length of 21 km to an altitude difference of 1,530 meters; its average gradient is thus 7.3 % (73/1000 or 0.073).

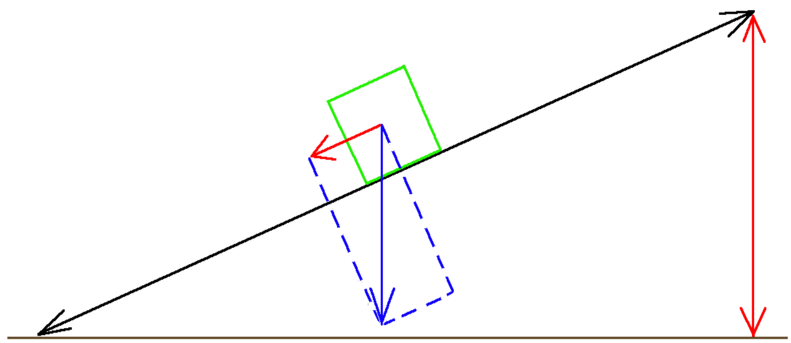
Warning ! Do not confuse this percentage with the angle formed by the road to the horizontal: a 10 % slope does not mean that this angle is 10 degrees!

From the mathematical point of view, the slope is the sine of the angle formed by the road to the horizontal, to distinguish the slope, the geometric sense, which is the tangent of the angle, even if the difference between these two values is negligible on the road network.

The component parallel to the road weight

What is the effect of the gradient on the movement of ground vehicles?

Due to the slope, the weight has a component parallel to the road whose intensity is proportional to the sine of the angle formed by the road with respect to the horizontal. This force is involved in the movement in the downhill, it opposes the movement in the rise.



Component of the weight on a road sloping.

Slope and traction

The traction force is defined as the force exerted on contact with the ground to create or maintain the movement of land vehicles.

The slope percentage immediate information on the traction force to be available to maintain the upward movement.

Indeed, if we neglect the action of both resistive forces (rolling resistance, air resistance), the traction force must be strictly equal and opposite to the component of the weight parallel to the road if we want to maintain a constant speed.

Example 1: a 100 kg mass cycling (bike included) to overcome a rise of 10 % will apply a traction force of about 100 N, or 10 % of its weight (‘**g**’ ~ 10 $m.s^{-2}$).

Example 2: a 1,500 kilograms mass car that must cross a rise to 10 % will have a traction force of about 1,500 N, or 10 % of its weight (‘**g**’ ~ 10 $m.s^{-2}$).

Slope and engine brake force

A similar reasoning applies for the purpose of stabilizing the speed in descent, if one neglects the action of resilient forces.

Example: a 1,500 kilograms mass car cross an 10 % down must have an engine brake force (see issue ADILCA '*engine torque*') of about 1,500 N, or 10 % of its weight (‘**g**’ ~ 10 $m.s^{-2}$).

Gradient and braking force if a braking is required, the component of the weight parallel to the road is added to the braking force uphill, is subtracted from the braking force downhill.

Example: a 1,500 kg mass car with a braking force of 15,000 N in a horizontal way (see ADILCA folder 'braking force') provides, all other conditions being equal, a force braking of approximately 16,500 N in a 10 % rise, and about 13,500 N in a 10 % lowering ('**g**' ~ 10 $m.s^{-2}$).

Slope and work

The work in the physical sense of the term, is defined as the energy required to move a force.

Neglecting air resistance and rolling resistance, though more is assumed that the transmission efficiency of the bicycle is total, the work of a cyclist on a constant slope depends only on the distance traveled.

Example 1: if we consider a cyclist mass 100 kg (bicycle included), crossing a rise of 10 km to 10 % requires work about 1 MJ and therefore mobilizes equivalent muscular energy.

Example 2: if we consider a car mass 1,500 kg, crossing a rise of 10 km to 10 % requires work of about 15 MJ and therefore mobilizes equivalent motive energy.

Slope and power

The power is defined as the work done per unit of time.

Neglecting air resistance and rolling resistance, though more is assumed that the transmission efficiency of the bicycle is total power to the physical sense only depends on the speed with which the work is accomplished.

Example 1: if we consider a cyclist mass 100 kg (bicycle included), the crossing of a rise to 10 % at a constant speed of 4 $m.s^{-1}$ (9 mph) mobilizes an output of about 400 W, a value well beyond the physical capacity of a normal individual.

Indeed, although some very talented and well-trained cyclists are able to temporarily deliver a power of 450 W, it is assumed that an effort of endurance achieved by a normal individual should not mobilize a power greater than 100 W.

Example 2: If we consider a car mass 1,500 kg, the crossing of a rise to 10 % at a constant speed of 20 $m.s^{-1}$ (45 mph) mobilizes an output of approximately 30 kW (either 41 hp).

In reality, much greater power is required due to the rolling resistance and air resistance, the latter being proportional to the square of the velocity (see ADILCA files).

Altitude and Weather

Weather conditions (atmospheric pressure, temperature) vary with altitude. This is because the air density decreases gradually as one moves away from the ground. And when the density of air decreases, the temperature drops.

Considering the characteristics of a vertical column of air, it is found that 1000 meters, the air density is 12 % smaller than the sea level.

At 2,000 meters, this density is 21 % lower, and at 3,000 meters, it is 29 % lower than the level of the sea.

If the temperature of the air column is 20° C at sea level, it is 15° C at 1,000 meters altitude, 10° C at 2,000 meters and 5° C at 3,000 meters, etc.

For cyclists that evolve in the mountains, these factors are taken into consideration. The most immediate effect is the feeling of gasping for air as lungs fill more difficult.

In addition, in equal volumes, the weight of absorbed air with each breath is reduced. Decreases muscle performance accordingly, especially as the body must also compensate for lower ambient temperature.

Therefore, except for acclimatization to increase the number of red blood cells responsible for precious oxygenation, physical efforts in the mountains are always very demanding and we must be careful to compare sport performances in altitude with those completed at of the sea.

Of course, these physical realities have the same consequences on the operation of automobile engines since they need air to burn fuel (see ADILCA folder).

(*)*The Alto de Angliru, located in the Spanish province of Asturias, is considered one of the steepest mountain passes in the world with 23 % crossings.*

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SOME RELATIONSHIPS BETWEEN QUANTITIES

Weight :

$$\mathbf{P = M \cdot g}$$

P : weight, expressed in **N**
M : mass, expressed in **kg**
g : gravitational acceleration, expressed in $\mathbf{m.s^{-2}}$
(Earth : g = 9.8 $m.s^{-2}$)
consistency of the units : **P** = kg . $m.s^{-2}$ = **N**

Example : calculate the weight of a 1,500 kg mass car :

$$P = 1{,}500 \times 9.8 = 14{,}700 \text{ N}$$

Slope :

$$\mathbf{\alpha = H / L}$$

α : slope, dimensionless ;
H : height, expressed in **m**
L : lenght, expressed in **m**
consistency of the units : **α** = m^{+1} . m^{-1} = dimensionless.

Example : calculate the slope of a 10 kilometers (6.2 miles) road with a 1,000 meters (3,000 feet) height :

$$\alpha = 1{,}000 / 10{,}000 = 0.1 = 1/10 = 10\ \%$$

Component of the weight :

$$\mathbf{F = M \cdot g \cdot \alpha}$$

F : component of the weight, expressed in **N**
M : mass, expressed in **kg**
g : gravitational acceleration, exprimée en $\mathbf{m.s^{-2}}$
α : slope, dimensionless ;
consistency of the units : **F** = kg . $m.s^{-2}$ = **N**

Example : calculate the component of the weight of a 1,500 kg (3,300 lb) mass car climbing a 0.1 gradient slope (g = 9.8 $m.s^{-2}$) :

$$F = 1{,}500 \times 9.8 \times 0.1 = 1{,}470 \text{ N}$$

Traction work :

$$\mathbf{E = F \cdot D}$$

E : work, expressed in **J**
F : component of the weight, expressed in **N**
D : distance, expressed in **m**
consistency of the units : **E** = N . m = $kg.m^{+1}.s^{-2} . m^{+1} = kg.m^2.s^{-2}$ = **J**

Example : calculate the traction work of a 1,470 N force which travels upon 10 kilometers (6.2 miles) :

$$E = 1{,}470 \times 10{,}000 = 14{,}700{,}000 \text{ J}$$

Gravitational energy :

$$\mathbf{E = M \cdot g \cdot H}$$

E : energy, expressed in **J**
M : mass, expressed in **kg**
g : gravitational acceleration, expressed in $\mathbf{m.s^{-2}}$
H : height, expressed in **m**
consistency of the units : **E** = $kg . m^{+1}.s^{-2} . m^{+1} = kg.m^2.s^{-2}$ = **J**

Example 1 : calculate the gravitational energy of a 1,500 kg (3,300 lb) mass after a 1,000 meters height (**g** = 9.8 $m.s^{-2}$) :

$$E = 1{,}500 \times 9.8 \times 1{,}000 = 14{,}700{,}000 \text{ J}$$

Example 2 : calculate the gravitational energy of a 1,500 kg (3,300 lb) in free fall from a 1,000 meters height (**g** = 9.8 $m.s^{-2}$) :

$$E = 1{,}500 \times 9.8 \times 1{,}000 = 14{,}700{,}000 \text{ J}$$

Power absorbed by slope :

$$\mathbf{B = F \cdot V}$$

B : power, expressed in **W**
F : component of the weight, expressed in **N**
V : speed, expressed in $\mathbf{m.s^{-1}}$
consistency of the units : **B** = $kg.m^{+1}.s^{-2} . m^{+1}.s^{-1} = kg.m^2.s^{-3}$ = **W**

$$B = M \cdot g \cdot H / T$$

B : power, expressed in **W**
M : mass, expressed in **kg**
g : gravitational acceleration, expressed in $m.s^{-2}$
H : height, expressed in **m**
T : duration, expressed in **s**
consistency of the units : **B** = $kg \cdot m^{+1}.s^{-2} \cdot m^{+1} \cdot s^{-1} = kg.m^{2}.s^{-3}$ = **W**

Example 1 : calculate the power absorbed by the slope, the component of the weight being a 1,470 N force at a speed of 20 $m.s^{-1}$ (45 mph):

B = 1,470 x 20 = 29,400 W

Example 2 : calculate the power absorbed by the ascension of a 1,500 kg mass upon a 1,000 m height in 10 minutes:

B = 1,500 x 9.8 x 1,000 / 600 = 24,500 W

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