THE CORIOLIS FORCE

I. THE LAWS OF NEWTON

II. CORIOLIS FORCE: THE TRUE DEFINITION

III. CORIOLIS FORCE: THE CALCULATION MODE

IV. BIBLIOGRAPHY

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I. THE LAWS OF NEWTON

The general laws of movement were discovered and formulated by the English mathematician and physicist Isaac Newton (1642 - 1727).

These laws are universal and allow you to describe any form of movement.

These laws read as follows:

Principle of rectilinear inertia

'*A moving mass on which no force acts, describes a perfectly rectilinear trajectory.*'

The concept of force stems from this principle.

Force concept

'*A force refers at any cause which deviates the trajectory of a mass.*'

Principle of reciprocity

'*Any mass subjected to a force, responds by a reciprocal action of equal intensity, but of opposite direction.*'

How do these laws apply in the case of a vehicle that travels on the surface of the Earth, and how can we define the concept of Coriolis force?

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II. CORIOLIS FORCE: THE TRUE DEFINITION

Have you ever tried to play "clay pigeon shooting" during a carnival? It is already no easy task to reach a motionless target, then what about a moving target…

Let us think of the same thing on the scale of Earth with, for instance, a missile fired from a helicopter in hovering flight over the North Pole (the pilot could see Earth turning under his feet, through the cockpit), this missile aiming a motionless target located at the equator [1].

To approach the problem correctly

The problem is outlined like this: observed from the helicopter and right after being thrown, the missile goes on a straight trajectory (Newton's *principle of inertia*), whereas the target, although motionless, continuously follows a rotary motion that coincides with the one of the Earth, because the Earth rotates around itself.

This time seen from Earth, the missile seems deviated from its usual trajectory, as if it were subjected to a force. That is the Coriolis force.

Who was Coriolis?

Gaspard Coriolis (1792 - 1843) was a French military engineer.

His most prominent work consisted in solving artillery and ballistics problems which, at the beginning of the 19th century, became more difficult with the progresses of long-range shots (several tens of kilometers).

There were not these problems at the time of Newton, 150 years earlier, and there is no more today, due to laser-guided bombs.

From Paris to the equator

What would the trajectory of the missile be if, instead of being fired from a helicopter at the North Pole, it was fired from the ground in Paris?

As Earth has a spherical shape and turns on itself, every point of the Earth's surface has a peripheral speed that is proportional to the distance from the Earth's rotational axis, in other words: inversely proportional to its latitude.

This speed, of course nil on the poles (latitude 90°), reaches 300 m.s^{-1} in Paris (49° north latitude) and 464 m.s^{-1} at the equator (latitude 0).

If we observe the shooting from a helicopter in hovering flight relative to the Sun, we notice that the missile describes an oblique trajectory composed by two perpendicular velocity vectors of constant modulus.

One vector corresponds to the peripheral speed of the place from where the shot is fired, it is pointing to the east, i.e. in the same direction as the Earth's rotation; the other corresponds to the speed of the missile, it is pointing to the south, which means towards the target located at the equator.

This wouldn't impact the accuracy of the shooting if the missile and the target had strictly identical peripheral speeds, yet it is precisely not the case. Quite the opposite actually, because the difference is growing as the missile gets closer of its target.

Therefore it is just as if the target shied away from the missile.

Seen from the Earth, the missile seems deviated from its actual trajectory, as if it were driven by a force. That is the Coriolis force.

The other "Coriolis effect"

The Coriolis phenomenon not only affects the trajectory of projectiles, but also that of a mass freefalling to Earth, for the same reason, which is the difference of peripheral speed between the start point and the final point of the fall.

This effect, called 'vertical Coriolis effect' to distinguish it from the 'horizontal Coriolis effect' described above, is nil for a mass dropped at the vertical of a pole, in the axis of rotation of Earth. On the other hand, it is maximal at the equator. Between the pole and the equator, it depends on the latitude of the place and on the height of the fall.

To understand this phenomenon, just let a ball roll on a table. As soon as the ball arrives at the edge of the table, it falls. However, it doesn't reach the floor exactly vertically below the edge of the table, but slightly close by, because of its initial speed. And the higher its initial speed, the greater the difference.

The vertical Coriolis effect is a comparable logic. Imagine the same ball dropped without initial speed from the Eiffel Tower, in Paris, disregarding the air resistance and the turbulences, as if the ball fell in a perfect vacuum.

Because of the Earth's sphericity and rotation, the peripheral speed of the top of the Eiffel Tower is of 0.015 m.s^{-1} superior to that of the foot.

This velocity vector being perpendicular to the trajectory of the fall, everything happens as if the ball fell with a nil horizontal speed at the beginning of the fall, but in constant increase as it gets closer to the ground.

For the observer, the trajectory described by the ball bends as it falls, as if it were driven by a transverse force. Thus it reaches the ground not exactly vertically below the

point from where it was dropped, but nearby. Given the direction of rotation of Earth, this deviation is oriented eastwards.

Force or effect?

The Coriolis force is an apparent force. Like all apparent forces (there are only three: the inertial force, the centrifugal force and the Coriolis force!), it is a fictitious force which has no real existence.

That's why the physicists use the words '*Coriolis effect*' rather than of force, in order to distinguish the effect from the cause. The Coriolis effect, by the way predicted by Newton ('*Principia*', published in London in 1687), is not due to a force, it is only caused by the rotary motion of the Earth [2].

So there is no Coriolis effect if a planet is free of rotation on itself [3].

A negligible effect

A set of calculations shows that any mass dropped from the Eiffel Tower in Paris (1,000 ft high, 8-second free-fall) is only deviated by the Coriolis effect by less than ten centimeters when it touches the ground (disregarding the air resistance and the turbulences, as if the mass fell in a perfect vacuum)!

We deduce that the Coriolis effect, whether horizontal or vertical, is negligible for a short distance or duration.

Some misconceptions on Coriolis effect…

The tides are permanent oscillations of the liquid masses due to the attraction of the Moon and of the Sun. Nevertheless, these phenomena are only perceivable on a large scale and on huge extents. So it is absolutely impossible to observe them or to feel their effects on the shore of a lake, let alone on the edge of a swimming-pool.

Similarly, the Coriolis effect is far too weak to play any role in the water flow in a tub or a sink: it would require a large round lake covering an area of several square kilometers, provided with a central funnel-shaped plughole, to appear significantly.

The Coriolis effect doesn't affect the movement of cars, its magnitude being incommensurate with the various parameters that influence the trajectory of land vehicles (road layout, banking, crosswind…). Even if we consider rally cars that take off from the ground when hitting bumps, their jumps are too short and too brief to be affected in any way by this effect.

In other words: the Coriolis effect is too weak to deflect the trajectories of cars!

Once again, a reference story!

Then the reference frame's theory (see ADILCA files) applies to the problem of the trajectory of a missile fired from Earth:

- here, the absolute reference frame is the Sun (said *inertial* or *Galilean*, referring to Galileo Galilei, the first to raise the issue of a reference frame in the description of a movement), reference frame to observe both the Earth's rotation motion and the actual trajectory of the missile;

- here, the relative reference frame is the Earth (said *non inertial* or *non Galilean*), reference frame to observe the trajectory of the missile, as if the Earth had stopped turning on itself.

In the reference frame of the Sun, the Coriolis force doesn't exist. The Coriolis effect can only appear in a relative reference frame (most of the time: the Earth), hence excluding any description of the rotary motion of the reference frame. Both descriptions cannot mix or be overlaid.

The Coriolis force: a novel definition!

From the above, we can draw this novel definition of the Coriolis force:

"*Coriolis force refers to the transverse force that should be exerted on any kind of projectile, or on any mass in free-fall, if Earth stopped turning on itself, in order to obtain the same trajectory as the one it seems to describe seen from Earth.*"

Note the hypothetical character of this force, resulting from the use of the conditional ('*the force that should be exerted*...') and the two conditions required by this definition:

1. First: Earth should stop turning on itself.

2. Second: it is impossible to exert any force on any projectile once launched, or on any mass falling in free fall.

Conclusion

Like centrifugal force and inertial force, the Coriolis force is an imaginary force. The Coriolis phenomenon, incorrectly associated with the concept of force, comes down to a simple effect caused by an optical illusion.

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III. CORIOLIS FORCE: THE CALCULATION MODE

1. Horizontal Coriolis effect

Let us figure out the magnitude of the Coriolis effect exerted on a car weighing 3,300 lb (1,500 kg) and circulating near Paris (49° north latitude) at the speed of 65 mph (30 m.s^{-1}) on a south-north, or north-south track [4]:

$$F = 2 M . V . \omega . \sin \theta$$

F: Coriolis effect (in **N**)
M: mass of the car (in **kg**)
V: speed of the car (in **m.s^{-1}**)
ω: rotation speed of the reference (Earth: ω = 7.27 x 10^{-5} **rad.s^{-1}**)
θ: latitude considered location (Paris: 49° north latitude; sine 49° = 0.75)

consistency of the units [5]: $F = kg . m.s^{-1} . s^{-1} = kg.m.s^{-2} = N$

$$F = 2 \text{ x } 1,500 \text{ x } 30 \text{ x } 7.27 \text{ x } 10^{-5} \text{ x } 0.75 = \mathbf{5 \text{ N}}$$

How to interpret this result? On an automobile, such a small effect can only occur if the following conditions are met:

- the roadway is flat, without any banking;

- it describes a perfect straight line that is south-north or north-south oriented;

- there is no crosswind;

- the car body is in perfect condition, without any accessories, and the windows are fully closed;

- the front and rear wheel axles are not affected by any parallelism issue or any misalignment;

- each of the four tires has the same state of wear, the same pressure and is equally loaded.

These conditions are obviously impossible to meet!

But if that were the case, the Coriolis effect being perpendicular to the trajectory, the car would give the impression of "pulling to the right" in the Northern hemisphere, and of "pulling to the left" in the Southern hemisphere.

Be careful! It is an effect and not a force. In other words, the car would not be driven

by any transverse acceleration. So the driver would not feel anything particular, except for a visual impression: he would simply notice that the car, left to itself, gradually moves away from the ideal trajectory…

So the Coriolis force corresponds to the magnitude of the guiding force (see Adilca file named "guiding force"), that would be triggered by the driver to keep the car in line with the road axis, if this effect was perceptible [6].

Independent of the longitude, the Coriolis effect is proportional to the latitude: it would be equal to 4 N if, all else being equal (mass, speed, trajectory), the car circulated near Athens or Lisbon (38° north latitude), and 6 N near Helsinki or Oslo (60° north latitude).

To assess the magnitude of the Coriolis phenomenon, let us calculate the magnitude of the guiding force that should be triggered by a driver in a car weighing 3,300 lb (1,500 kg), wishing to make a complete turn of the Paris ring road (22 mi, 35 km), at the speed of 45 mph (20 m.s^{-1}), assuming a flat and perfectly circular ring road:

$$F = M . V^2 / R = M . V^2 . [C / (2 \pi)]$$

F: guiding force (expressed in **N**)
M: mass of the car (expressed in **kg**)
V: speed of the car (expressed in **m.s^{-1}**)
R: trajectory radius (expressed in **m**)
C: circumference of the Paris ring road (expressed in **m**)

consistency of the units [5]: $F = kg . (m.s^{-1})^2 . m^{-1} = kg . m^{+2}.s^{-2} . m^{-1} = kg.m.s^{-2} = $ **N**

$$F = 1,500 \times 20^2 / (35,000 / 6.28) = 600,000 / 5,600 = 110 \text{ N}$$

No one is sensible to this force, but the magnitude is yet more than 20 times greater than the Coriolis effect assessed above!

2. Coriolis acceleration

As we have just written, the Coriolis force does not generate any acceleration since it is a fictional force. But then, what is the so-called 'Coriolis acceleration'?

The car 'pulling a side', the so-called 'Coriolis acceleration' is the transverse acceleration that it must undergo to cancel this effect, if the driver wants to stay in the axis of the road and maintain his initial course.

This acceleration is generated by the *guiding force*, itself induced by the pivoting of the steering wheels, at the initiative of the driver (see ADILCA file '*guiding force*').

If the mass of the car and the intensity of the Coriolis effect are known, it is easy to calculate this acceleration, by virtue of the fundamental principle of the dynamics enunciated by Isaac Newton [7]:

$$Y = F / M$$

Y : transverse acceleration, expressed in **m.s^{-2}**
F : Coriolis effect, expressed in **N**
M : mass, expressed in **kg**
consistency of the units : $Y = kg^{+1}.m^{+1}.s^{-2} . kg^{-1} = $ **m.s^{-2}**

Exemple : let's go back to the previous data (car weighing 3,300 lb [1,500 kg] and circulating near Paris at the speed of 65 mph, Coriolis effect of 5 N) and calculate the transverse acceleration that this car must undergo, if the driver wants to stay in the axis of the road:

$$Y = 5 / 1,500 = 0.003 \text{ m.s}^{-2}$$

This acceleration is of an intensity about three thousand times lower than that exerted by the Earth (9.8 m.s^{-2}), and still about twice as weak as that exerted by the Sun (0.006 m.s^{-2}) [8]!

3. Reciprocal action

Recall this principle stated by Isaac Newton:

'*Any force exerted on any mass causes a reciprocal action of equal but opposite intensity.*'

No real force, no reciprocal action. The Coriolis force being an imaginary force, there is no reciprocal action.

Hence, this principle is applicable only if the driver needs the *guiding force* to counteract the Coriolis effect described above (see ADILCA file '*guiding force*').

In that case, the Newton's principle applies, but it obviously applies to the guiding force, a real force, and certainly not to the Coriolis force, an imaginary force.

As the guiding force exerts at the periphery of the tires in contact with the ground, the reciprocal action is that which the car exerts on the Earth by means of tires.

As stated by Isaac Newton, this reciprocal action is of the same intensity as the guiding force, but in opposite direction. It is therefore very easy to calculate, from the following relation:

$$A = - F$$

A: reciprocal action, expressed in **N**
F: guiding force, expressed in **N**

(the sign [–] specifies the spatial orientation of this reciprocal action)

For example, let's calculate the reciprocal action the car exerts on the Earth when the driver requests a guiding force of 5 N (as calculated above):

$$A = -5 \, N$$

Obviously, this reciprocal action is of a very weak intensity, especially since it is exerted on a huge mass, that of the Earth. So, it is insufficient to disturb the stability of the Earth and its rotational movement.

Indeed, the principle of reciprocal action can be seen as a balance of power between two masses: the mass of the car (1,500 kg) and that of the Earth (6 x 10^{24} kg). A ratio in favor of the Earth, to the detriment of the car.

4. Vertical Coriolis effect

Let us calculate the deviation of trajectory caused by Coriolis effect on an object freefalling from the top of the Eiffel Tower in Paris:

$$D = 2/3 \, \omega \, . \, \text{cosine} \, \theta \, . \, (2 \, h^3 \, / \, g)^{1/2}$$

D: deviation of trajectory due to Coriolis effect (expressed in **m**)
ω: rotation speed of the reference (Earth: ω = 7.27 x 10^{-5} rad.s^{-1})
θ: latitude of the considered location
(Paris: 49° north latitude; cosine 49° = 0.66)
h: height of the fall (Eiffel Tower: 1,000 ft, 320 **m**)
g: gravitational acceleration of the considered location (Paris: g = 9.8 **m.s^{-2}**)

consistency of the units [5]: $D = s^{-1} . (m^{+3} . m^{-1}.s^{+2})^{1/2} = s^{-1} . (m^{+2}.s^{+2})^{1/2} = s^{-1} . (m^{+1}.s^{+1}) = m$

$$D = 2/3 \times 7.27 \times 10^{-5} \times 0.66 \times (2 \times 320^3 \, / \, 9.8)^{1/2}$$

$$D = 2/3 \times 7.27 \times 10^{-5} \times 0.66 \times 2^{1/2} \times 320^{3/2} \, / \, 9.8^{1/2}$$

$$D = 2/3 \times 7.27 \times 10^{-5} \times 0.66 \times 1.4 \times 5,700 \, / \, 3.13$$

$$D = 8,075 \times 10^{-5} = 0.08 \, m$$

How to interpret this result? The deviation of trajectory attributed to Coriolis effect can only appear if the following conditions are met:

- the object has a perfectly homogeneous mass, a strictly spherical shape and a perfectly smooth outer surface;

- the object, during its fall, slides in a clean air, without generating any aerodynamic turbulence;

- there is no crosswind…

All conditions obviously impossible to meet!

Actually, numerous experiments led in ideal atmospheric conditions have shown that the exact localisation of the impact point, on the ground, of a steel ball after such a fall is only accurate to within ± 0.15 meters.

Thus, the uncertainty due to the experimental conditions thus has an influence on average three times greater than the Coriolis effect!

(1) *The influence of winds and gravity on the trajectory of the missile is neglected. A helicopter in hovering flight is assumed to be independent of the movement of the atmosphere that covers the Earth, making it an absolute reference frame.*

(2) *However, there would be no horizontal Coriolis effect if the Earth had the shape of a cylinder, nor of vertical Coriolis effect if it had the shape of a flat disk.*

(3) *Such a planet does not exist in the Solar System.*

(4) *The horizontal Coriolis effect always appears on the same side, regardless of the direction, it is only reversed when changing hemisphere.*

(5) *The angle measurements, the trigonometric ratios, the coefficients etc. are dimensionless quantities that don't affect the units with which they are combined.*

(6) *The guiding force intended to cancel the Coriolis effect, the driver should steer in the opposite direction, in other words to the left in the northern hemisphere, to the right in the southern hemisphere.*

(7) *If the mass of the car and the intensity of the Coriolis effect are known, it is easy to calculate this acceleration thanks to the fundamental principle of the dynamics stated by Isaac Newton:* [$Y = F / M$] *where* **F** *is the effect Coriolis (expressed in* **N***) and* **M** *car mass (expressed in* **kg***).*

(8) *In the northern hemisphere, the Coriolis effect and the solar attraction combine when the car heads north at sunrise, when the car heads south at sunset. Conversely in the southern hemisphere. The attraction of the Moon, 200 times weaker than that of the Sun, is obviously negligible.*

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