# THE CENTRIFUGAL FORCE

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# I. THE LAWS OF NEWTON

The general laws of movement were discovered and formulated by the English mathematician and physicist Isaac Newton (1642 - 1727).

These laws are universal and allow you to describe any form of movement.

Being a circular movement, these laws read as follows:

## Principle of rectilinear inertia

'*A moving mass on which no force is acting, goes on a straight direction.*'

The concept of force stems from this principle.

## Force concept

'*A force refers at any cause able to deflect a path.*'

## Principle of reciprocity

'*Any mass subjected to a force, responds by a reciprocal action of equal intensity, but of opposite direction.*'

How do these laws apply in the case of a land vehicle in motion on a circular path, and how can we define the concept of centrifugal force?

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## II.  CENTRIFUGAL FORCE: THE TRUE DEFINITION

The concept of centrifugal force rarely comes with an instruction manual: like the Coriolis force and the inertial force, centrifugal force belongs to the category of fictitious forces, also called apparent forces or pseudo-forces.

Why are they called this way? The reason is that these forces can only exist in the context of imaginary descriptions. In effect, these forces do not really exist.

Here is a selection of various observations, experiments or demonstrations supposedly proving the existence of centrifugal force, followed by the correct interpretations.

**Reminder ...**

A *reference frame* refers to a coordinate system from which the characteristics of the motion of a mass can be measured [1].

According to the *principle of inertia* of Isaac Newton, a moving mass on which no force is acting describes a *rectilinear path*. Hence this definition: a *force* refers to any cause that deflects the *path* of a mass.

*Centrifugal* means '*which takes away from the center*'.

Thus, according to these definitions, a force qualified as centrifugal should be able to push a mass away from a center on a radial path, i.e. in the direction indicated by the extension of a radius.
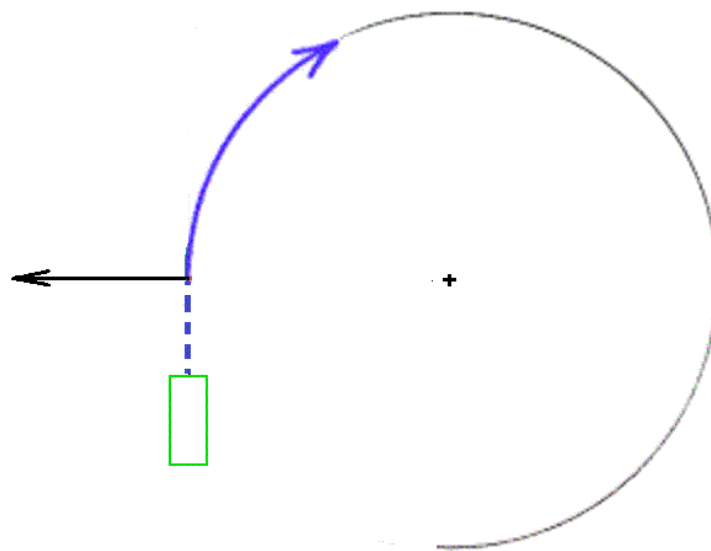
When a car enters a circular path (blue arrow), the centrifugal force should act in the direction indicated by the extension of the radius of the curve (black arrow). Such a move never happens.

**The circular motion ...**

If we observe a car crossing a curve, there are only three possibilities, but the car never moves away on a radial direction:

1. The driver ignores the turn, the car goes straight (dashed black arrow). As Newton said, it is the normal path of a mass because no transverse force is acting.

2. The driver places the car on a circular path and normally crosses the curve (green arrow), which is the case most of the time.

3. The driver goes off of the road (blue arrow). However, the car was deflected from its rectilinear path. Incompletely deflected, but deflected anyway. That brings us back to the case number 2.

Note that centrifugal force never appears in any case at any time.
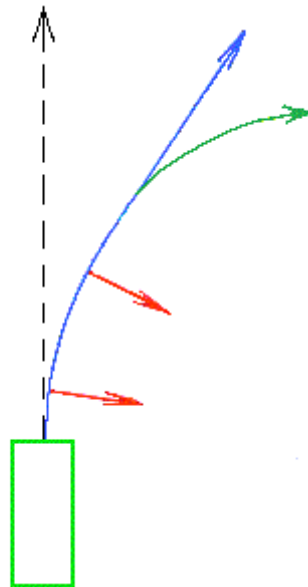
The three possible trajectories:
1. The driver ignores the turn and goes straight (black dotted arrow).
2. The driver normally crosses the curve (green arrow).
3. The driver goes off the road (blue arrow).
The cause that has deflected the rectilinear path is shown in red.
There is no centrifugal force.
(Be careful not to mix forces and trajectories!).

In a way, this experiment validates Isaac Newton's principle of rectilinear inertia. When a car maintains a rectilinear path, it is because it is left to itself, in other words, when the car is not subject to any force, and certainly not because of any force, centrifugal or otherwise.

### … and its cause

The previous experiment also validates that the circular path of the car is due to a force. Where, why and how is this force exerted?

The rotation of the steering makes pivot the steering wheels, so a transverse force is exerted on the tires in contact with the ground, it is the guiding force (see ADILCA file '*guiding force*'). This force is the sole cause of circular motion.
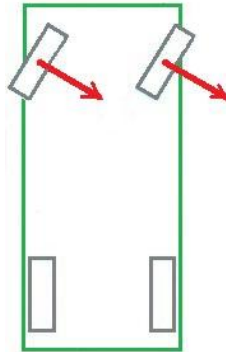
Note: the guiding force is oriented towards the center of the curve but is not centripetal: it does not act at a distance and does not act on the center of mass of the car (see ADILCA file '*centripetal force*').

### The object on the hood ...

What is centrifugal force? To understand it, an experiment is necessary.

Let us put an object on the hood of a car (e.g. a 'traffic cone'). The experiment consists in setting the car in motion, first in a straight line then a turn.

The so-called 'traffic cone' experiment

Now observe this experiment from a window or a balcony: as soon as the driver steers the steering wheels, the guiding force deflects the car from the straight line.

As the hood is a smooth area, this force cannot be transmitted to the object, which keeps its rectilinear direction and falls to the ground. The object in question is not driven by any force, it is simply left to itself.

Seen from the inside of the car, the object crosses the field of view before being ejected, as if pulled away from the car by an apparent force.

This apparent force is called centrifugal force, but this force does not exist since the traffic cone is simply delivered to itself.

## Two reference frames

These two observations are contradictory and incompatible because it is important to distinguish between two reference frames:

1. The Earth is the reference frame to describe the path of the traffic cone if the move is observed from a window or a balcony, so this description takes place in an absolute reference frame [2].

1. The car is the reference frame to describe the path of the traffic cone, if the move is observed from the driver or passenger seat, so this description takes place in is a relative reference frame.

Why is the car reference frame qualified as relative?

The motion of a land vehicle only exists in relation to the Earth, it does not exist in the reference frame of the car. This means that, in such a reference frame, the vehicle must be considered as motionless. Of course, it is forbidden to mix the two descriptions.

Now we can analyze all the other experiments supposed to prove the existence of the centrifugal force.

## The mascot hanging from the mirror ...

Let's get in a car with a mascot hanging from the mirror and observe it.

In a straight line at constant speed, the mascot shows the vertical. When the car makes a turn, the mascot tilts to the side, as if driven by an apparent force. This apparent force is centrifugal force.

But in the reference frame of Earth, the mascot is only deflected from an initially straight line: the guiding force is exerted first on the tires of the steering wheels and is then transmitted to the chassis, to the bodywork and all the car accessories; it finally reaches the mascot through the mirror after which it hangs. Hence its inclination.

Mascot hanging from the mirror

The apparent force which, from the passenger's point of view, seems to move the mascot, is therefore only an optical illusion. In fact, this force does not exist.

**The movement of passengers ...**

When the car makes a turn at top speed, the passenger of a car feels pressed against the edge of the seat or against the door, as if driven by an apparent force. Where does this sensation come from? Is it the centrifugal force?

When the driver turns the wheel, the car is driven by the guiding force exerted on the tires of the steering wheels; this force is then transmitted to the chassis, the bodywork and all the accessories of the car.

The objects that are firmly attached to the body undergo this force fully and without delay, stowage consisting precisely in enabling the body to communicate this force.

But this is not the case for the passengers who, though seated on their seats, still retain some freedom of movement. When the car starts to turn, the passengers maintain a straight path, just like the mascot in the previous experiment, and they do so until the edge of the seat, the door or the body transmits the guiding force to them.

It is not therefore the centrifugal force that the passenger's experience, but simply the guiding force exerted on the car and transmitted by the seat, seatbelt or part of the body of the car.

**The movement of luggage ...**

Let's take a look at luggage placed in the trunk or objects on the rear shelf.

The explanation is the same as for passengers: when the car turns, the movement of luggage placed in the trunk or objects on the rear shelf is only apparent in relation to the car.

In fact, objects that are not perfectly secured maintain a straight path as long as no part of the body can transmit a guiding force to them.

Conclusion: the centrifugal force does not exist, neither in the cockpit, nor on the rear shelf, nor in the trunk!

### Centrifugal force: the true definition

All these observations, all this reasoning, all these deductions lead us to two original and unpublished definitions of centrifugal force:

*'In the reference frame of the car, centrifugal force is the imaginary force that would have to be exerted on the center of mass of the passengers and luggage of a stationary car to see them driven by a motion identical to that observed in reality when the car is driven by the guiding force.'*

There are three fundamental requirements to these definitions:

1. The car must be stationary.

2. This force is hypothetical, as clearly stated by the conditional: *'the force that would have to be exerted'*...

3. The point at which to exert this force: it is technically impossible to exert any force directly on the center of mass of any mass... This requirement alone would suffice to prove the unreality of centrifugal force.

These are sufficient reasons to assert that centrifugal force does not exist!

This definition makes it clear that the reference here is the car reference system. Which brings another question: could centrifugal force appear in the Earth reference system?

### The rolling motion ...

Observe a Citroën 2CV full turn. Because of the soft suspension, the outside wheels are compressed while the weight on the inner wheels is released. This phenomenon is called the 'roll'.

Why does the car behave this way? To deflect the car from its initially straight path, the driver had to apply a transverse force called guiding force. This force is exerted on the steering wheels in contact with the ground, but not on the center of mass.

It is therefore the height of the center of mass that explains the rolling motion: the car turns in on itself in a transverse plane. Under the effect of the guiding force, the car simply acts like a person unbalanced by the carpet being pulled under her feet.

If the guiding force was exerted directly on the center of mass, there would be no roll and the car would take 'flat' corners. So, the rolling motion has nothing to do with the centrifugal force.

Conclusion: the centrifugal force does not exist, neither inside the car, nor outside!

### Both descriptions

However, the observation of the rolling motion allows two possible descriptions of the same phenomenon:

1. A real description, called 'dynamic', which describes all the movements of the car and their cause.

2. An imaginary description, called 'static' which considers that the car is immobile. In which case it would be necessary to imagine a force capable of creating a movement of artificial roll.

Now the concept of centrifugal force comes in:

*'In <u>statics</u>, centrifugal force is the <u>imaginary force</u> that would have to be exerted on the center of mass of a <u>stationary car</u> to create on the tires and suspensions an identical effect to that observed in reality when the car is driven by the guiding force.'*

The three previously stated requirements remain valid:

1. The car must be stationary.

2. This force is hypothetical, as clearly stated by the conditional: *'the force that <u>would</u> have to be exerted'*...

3. The point at which to exert this force: it is technically impossible to exert a force directly on the center of mass of a car... This requirement would suffice to prove that centrifugal force is not real.

In any case, the description of imaginary forces that could be exerted on the center of mass of stationary cars is of absolutely no interest, cars being machines designed for movement and intended to satisfy the needs of displacement.

### Reciprocal action: the Newton's third law

Could centrifugal force be regarded as the reciprocal action to guiding force?

Consider Newton's law:

*'Any mass on which a force is acting, responds by a reciprocal action equal in magnitude but opposite in direction.'*

The previous experiments clearly demonstrate that the circular motion of the car is due to the action of a single force, the guiding force.

We observed that the guiding force is exerted on the tires in contact with the ground. Logically, the reciprocal action can only occur at ground level, too.

In fact, when a driver activates the guiding force, the car performs a horizontal thrust at ground level, and this thrust should logically affect the Earth's rotational movement.

Fortunately, the effect is purely theoretical because the mass of the car is considerably lower than that of Earth, so that the car has no choice but to slide or register obediently on a circular path [3].

In addition, should the effect be noticeable, it would be negated by the divergent trajectories of the large number of vehicles on the road.

This famous reciprocal action therefore exists, but it has nothing to do with the concept of centrifugal force!

**The feelings of the passengers**

The misuse of the concept of centrifugal force led motorists to believe that they could feel the effects of an imaginary force, thus proving its existence.

Let us detail the mechanism of the circular motion: the guiding force is exerted on the tires in contact with the ground, it is then transmitted to the passengers via the wheels, the frame, the bodywork and the seats.

The principle of reciprocity then applies: the passengers are driven by the guiding force transmitted by the bodywork and the seats, they thus exert a reciprocal action on the seats and the bodywork, of equal intensity but of opposite direction.

Therefore, what the passengers feel is this reciprocal action, but certainly not the centrifugal force.

All this is very logical because it is obviously impossible to observe or feel the effects of an imaginary force.

**What is the correct formula?**

Does the famous formula **F = MV²/R** prove the existence of centrifugal force?

Let us first make sure it truly is a force.

According to the *International System of Units* (symbol: **SI**) – compulsory in most countries around the world, for instance in United States since 1964, in United Kingdom since 2004 – force is measured and expressed in kilograms-meters per second squared (symbol: **kg.m.s$^{-2}$**).

That is a derived quantity obtained by combining three fundamental quantities: mass and length and time.

The measurement obtained is the very definition of a *newton* (symbol: **N**), the international unit of force.

To check the consistency of this formula, consider how the different quantities introduced in this equation are combined: mass is expressed in kilograms (symbol: **kg**), speed is expressed in meters per second (symbol: **m.s$^{-1}$**) and the radius of the path is expressed in meters (symbol: **m**).

Let us combine these different quantities:

$$\mathbf{F = MV^2/R}$$

$$\mathbf{F} = kg \cdot (m.s^{-1})^2 \cdot m^{-1}$$

$$\mathbf{F} = kg \cdot m^{+2}.s^{-2} \cdot m^{-1}$$

$$\mathbf{F} = kg \cdot m^{+2-1}.s^{-2}$$

$$\mathbf{F = kg.m.s^{-2} = N}$$

There is no doubt, this formula is perfectly consistent, and therefore it expresses the measurement of a force.

A force indeed, but which one? Centrifugal or guiding? That is the question!



**FORMULE DE LA FORCE CENTRIFUGE**

Force centrifuge = $\dfrac{\text{Masse du véhicule} \times \text{Vitesse au carré}}{\text{Rayon de virage}}$

$$FC = \frac{MV^2}{R}$$

FORMATION DU CONDUCTEUR

BEPECASER   54.1

Correct formula, but not correct name!
(French Ministry of Transport official document)

**What force is it?**

What force are we dealing with?

We have already shown elsewhere that there were only two possibilities and two only:

- from the reference frame of the car, the centrifugal force is an apparent force and, obviously, can only be exercised on passengers and luggage but certainly not on the car;

- from the reference frame of Earth, the guiding force is a real force which is exerted on both the car and everything it contains.

How can we tell? How to distinguish the two? How to settle the debate?

The answer comes naturally by examining the various quantities introduced in the equation of the mass of the car, its speed and the radius of the curve.

Isn't it obvious?! These variables exist in the reference frame of Earth but they have no existence in the reference frame of the car!

To understand this significant nuance, try to imagine for a moment what the speed of the car or the radius of its path could be by reasoning exclusively in the reference frame of the car...

Evidently, this formula does not express the centrifugal force, but only the guiding force. The latter and the latter only is what it is all about.


**How to measure centrifugal force ...**

Can the intensity of centrifugal force be measured?

Yes! It is indeed quite possible to measure the intensity of an imaginary force, i.e. the intensity of a force that does not exist, but that would have to be resorted to if... Physicists love this kind of exercise!

However, with regard to centrifugal force, the usual approach is not correct, and here's why.

First things first: in science, as a rule, one should always check the origin of the quantity one is faced with, what it represents, and how it was obtained. This is what you may call a principle of traceability.

Before any calculation is carried out, a physicist must perform experiments, define benchmarks and make measurements. The process is what matters most.

Calculations only come next, but they are necessarily based on concrete measures, numerical values whose origin and meaning are certified – in short, quantities that really exist.

It is only later, thanks to a purely theoretical reasoning, that the physicist can transpose his reasoning to the study of an imaginary phenomenon.

In other words, to get to centrifugal force, which is an imaginary force, it is necessary to start from the guiding force, which is a real force. Indeed, there is no imaginary force without a real force. But the reverse is not true: the guiding force alone is enough to explain everything while the centrifugal force is always necessarily dependent on the guiding force.

It is therefore strictly forbidden to mention the centrifugal force without explaining where it comes from, what it represents and how it was obtained.

These details of what is in fact a very logical approach are often ignored or overlooked. To illustrate this, here is a concrete example.

### A concrete example ...

Take the example of a car with a mass of 3,300 lb (1,500 kg) which describes a circular path of radius 330 ft (100 m) at a speed of 45 mph (20 m.s$^{-1}$).

The famous formula, which is often used indiscriminately and of which we have detailed the applications, calculates the intensity of the guiding force **F** that is exerted on the tires of the car in contact with the ground:

$$F = M V^2 / R$$

$$F = 1,500 \times 20^2 / 100 = 6,000 \ N$$

The corresponding transverse acceleration is:

$$Y = V^2 / R$$

$$Y = 20^2 / 100 = 4 \ m.s^{-2}$$

It is only from this result that we can deduce the intensity of the centrifugal force **F'**, the famous force that would have to be exerted on the center of mass of the car, if it was stationary, in order to produce an effect comparable to that observed when the car is subjected to the guiding force.

But what formula should be used? The following one, and no other:

$$F' = - M Y = - F$$

The calculation is easy: to produce an effect comparable to what is observed in reality, a force of – **6,000 N** would be required!

In other words, to a guiding force of **6,000 N** in a real description, there is a centrifugal force of – **6,000 N** in an imaginary description!

In effect, the 'guiding force' and the 'centrifugal force' have the same modulus but yet, everything divides them:

- the formula used to calculate the intensity;

- the point of application (one of these two vectors has its origin at the periphery of the tire and the other at the center of mass);

- the direction (here, the often forgotten [–] sign is crucial, it shows that the spatial orientation of the centrifugal force, if such force existed, should be strictly opposite to that of the guiding force);

- last but not least: one of these two vectors is applied to a moving car and the other to a stationary car!

In short, these two vectors do not belong to the same description at all! Thus: the intensity of the centrifugal force is deduced from that of the guiding force, and it is never the other way round!

And the quantity supposed to prove the existence of centrifugal force results in fact in confusion with the guiding force!

**The inertial sensor ...**

Can a simple inertial sensor (also called lateral accelerometer) directly measure the intensity of centrifugal force?

Let us detail the principle of operation of this device: a block capable of sliding in a tube, is maintained at rest by two springs, but can nevertheless move along a slider in case of lateral acceleration of the car. The device is securely attached to the car body

Let us get back to the example of a car in a circle of radius 330 ft (100 m) at a speed of 45 mph (20 m.s$^{-1}$).

If the mass of the block is $10^{-2}$ kg, if the device is accurately calibrated, the cursor indicates a force of $4 \times 10^{-2}$ N, this is the force necessary to curve the direction of the block.

The fundamental relation of dynamics enables one to calculate the intensity of the transverse acceleration communicated to the block by the car:

$$Ɣ = F / M$$

$$Ɣ = 4 \times 10^{-2} / 10^{-2} = 4 \ \textbf{m.s}^{\textbf{-2}}$$

Note that this transverse acceleration is strictly identical to that of the car, since the sensor is a part of the car body and describes a circular path of the same radius.

As there is no motion without cause, it is deduced that the transverse acceleration of the block comes from the guiding force exerted on the car.

In other words, the inertia sensor measures the intensity of the guiding force, and its operating principle has nothing to do with the concept of centrifugal force.

### Newton and centrifugal force

Where does the concept of centrifugal force come from?

Let us remember Isaac Newton's writings published in London in 1687 ('*Philosophiæ Naturalis Principia Mathematica*'). Isaac Newton distinguishes two types of forces:

'*Materiæ vis insita est potentia resistendi, qua corpus unumquodque, quantum in se est, perseverat in statu suo vel quiescendi vel movendi uniformiter in directum.*'

'*Vis impressa est actio in corpus exercita, ad mutandum ejus statum vel quiescendi vel movendi uniformiter in directum.*'

Look at the translation by Andrew Motte published in London in 1729 ('*The Mathematical Principles of Natural Philosophy*'):

'*The* 'vis insita'*, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or of moving uniformly forward in a right line.*'

'*An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.*'

To which of these two categories does centrifugal force belong?

Experience has shown that there is no '*vis impressa*' exerted in the extension of a radius.

Consequently, what is called centrifugal force designates a '*vis insita*', which is not a force, but a principle, that of inertia applied to circular motion!

Look at the modern statement of this principle:

'*A moving mass on which no force acts, goes on a perfect straight direction.*'

This principle should have been called '*principle of rectilinear inertia*' to avoid some mistakes. The concept of force is deduced from this principle:

'*A force means any cause able to deflect a straight direction.*'

Obviously, there is no centrifugal force in Newton's writings.

### The advent of the statics...

The '*centrifugal force*' term was coined by Christiaan Huygens, Dutch mathematician (1629 - 1695) to describe the '*tendency*' of a mass in circular motion to move away from the center of rotation. A misleading name that Isaac Newton would correct a few years later: this '*tendency*' called '*vis centrifuga*' by Huygens is definitely renamed '*vis insita*' by Newton[4].

In the modern sense of the word, centrifugal force is an inertial force. This concept is inherent to the *statics*, a reasoning mode that we owe to Jean Le Rond d'Alembert, a French mathematician and physicist (1717 - 1783).

Statics is to reverse the reasoning by considering that any accelerated system could be described as being motionless, this inertia (in the real sense of the term) requiring a fictitious force to explain an apparent balance. This fictitious force is named inertial, or centrifugal.

In a work published in Paris in 1743 ('*Traité de Dynamique*'), D'Alembert thus delivers the key to static reasoning:

'*At all times, there would be an equilibrium between the forces actually acting on a set of moving material points, and the forces of inertia at various points in the system, if these were to act.*'

The word '*equilibrium*' could have led to believe that the two forces, equal and opposite, were acting at the same time on the same mass. But if that were the case, these two forces would neutralize each other. D'Alembert would then have used the expression '*destroy themselves*', since this is the term he uses elsewhere. Therefore, in D'Alembert's reasonning, the word '*equilibrium*' should be understood as meaning '*equivalence*'.

D'Alembert's sentence is therefore perfectly clear: the two forces are equal and opposite but do not act at the same time: real forces on one side (in dynamics), and fictitious forces on the other (in statics). Two perfectly separate descriptions. Besides, the use of conditional ('*there would be equivalence ... if they were to act*') clearly proves that inertial forces do not exist: they are just imaginary forces.

### The role of teachers...

Around the middle of the 19th century, the concept of inertial force, legitimized by the work of Gaspard Coriolis, a French military engineer (1792 - 1843, the inventor of the force that bears his name, see ADILCA file '*Coriolis force*'), aroused renewed interest among teachers, anxious to update programs in times of war.

What role have teachers been playing in this hoax?

While it is quite obvious that no physics teacher worthy of the name ever could mix up the origin of a phenomenon with its effects, the cause of a movement with its consequences, a real description with an imaginary description, that is still the way it goes.

And since most physics teachers remained confined in classrooms, lecture halls or laboratories, they focused their teaching on these famous imaginary concepts. Unconcerned about pragmatism, disconnected from reality, they forgot to issue the instruction manual. In short, as time and classes went by, imaginary descriptions irresistibly supplanted real descriptions.

### Conclusion

What is centrifugal force?

- In common language, centrifugal force refers to the principle of rectilinear inertia. A double confusion: a principle is not a force, and rectilinear inertia is not centrifugal.

- From a scientific point of view, the centrifugal force is an imaginary force to explain either an *apparent movement* described from a *relative reference frame*, or an *apparent balance* described in *statics*.

In short: centrifugal force does not exist. It is therefore wrong if this concept has been used to describe the phenomena observed in automobiles.

How then to explain the real movement? The normal trajectory of a car is straight by nature. To bend this trajectory, the driver must apply a transverse force called guiding force: this is the force exerted on the tires of the steered wheels in contact with the ground when the driver activates the steering control.

All other phenomena observed in automobiles, such as body roll of the car, movement of luggage or passenger sensations, etc., have clear, logical and rational explanations that have nothing to do with the concept of centrifugal force.

(1) *A coordinate system is made of three orthogonal axes (length, width, height), to which is associated a measurement of time (chronometer).*

(2) *The Earth is the absolute reference frame for describing the movement of land vehicles as they move in relation to the Earth. Warning: the same reference frame can be sometimes absolute (said inertial or Galilean), sometimes relative (said non-inertial or non-Galilean) according to the object of the study. Thus, the Earth becomes a relative reference frame to describe the movement of the planets of the solar system because they are moving around the Sun.*

(3) *The guiding force comes from the earth and acts on the car, the reciprocal action comes from the car and acts on the globe. These two forces are equal but their effect is inversely proportional to the mass on which they act, it is the second principle of Newton (dynamics relation [$F = M\ Y$] from where [$Y = F / M$ ]). If one compares a car of 2 metric tons and Earth (6 x 10$^{24}$ kg), the mass ratio is 1 to 3 x 10$^{21}$, 1 to 3,000 trillion.*

(4) *Christiaan Huygens gives neither explanation of the phenomenon nor theory of the physics, but a purely mathematical study based on the correlation between gravity, speed and radius of curve. Later, Isaac Newton filled this gap with the principle of rectilinear inertia and the concept of centripetal force.*

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# III. CENTRIFUGAL FORCE: THE CALCULATION MODE

## 1. Guiding force

$$F = M \cdot V^2 / R$$

**F**: guiding force, expressed in **N**
**M**: mass, expressed in **kg**
**V**: speed, expressed in **m.s$^{-1}$**
**R**: curve radius, expressed in **m**

Consistency of the des units: $F = kg \cdot (m.s^{-1})^2 \cdot m^{-1} = kg \cdot (m^2.s^{-2} \cdot m^{-1}) = kg.m.s^{-2} = N$

Example 1: calculate the guiding force acting on the tires which maintains a car of 1,500 kilograms (3,300 lb) on a circular path of 100 meters radius (330 ft) at a speed of 20 meters per second (45 mph):

$$F = 1,500 \times 20^2 / 100 = 1,500 \times 400 / 100 = 6,000 \textbf{ N}$$

By virtue of the principle of reciprocity, the tires of the car exert a thrust on the ground, of equal intensity but in opposite direction.

Example 2: calculate the guiding force acting on a passenger weighing 100 kilograms (220 lb) when the car describes a circular path of 100 meters radius (330 ft) at a speed of 20 meters per second (45 mph):

$$F = 100 \times 20^2 / 100 = 100 \times 400 / 100 = 400 \textbf{ N}$$

By virtue of the principle of reciprocity, the passenger exerts a thrust on the seat and the body of the car, of equal intensity but in opposite direction. This thrust is perfectly felt by the passenger, it has been confused with the centrifugal force.

## 2. Transverse acceleration

$$Y = F / M$$

**Y**: transverse acceleration, expressed in **m.s$^{-2}$**
**F**: guiding force, expressed in **N**
**M**: mass, expressed in **kg**

Consistency of the units: $Y = kg.m.s^{-2} \cdot kg^{-1} = \textbf{m.s}^{-2}$

Example: calculate the transverse acceleration of a car with a mass of 1,500 kilograms (3,300 lb) describing a circular path of 100 meters radius (330 ft) at a speed of 20 meters per second (45 mph):

$$Y = 6,000 / 1,500 = 4 \textbf{ m.s}^{-2}$$

## 3. Reciprocal action

$$A = - M . V^2 / R$$

**A**: reciprocal action, expressed in **N**
**M**: mass, expressed in **kg**
**V**: speed, expressed in **m.s$^{-1}$**
**R**: curve radius, expressed in **m**

Consistency of the des units: **A** = kg . (m.s$^{-1}$)$^2$ . m$^{-1}$ = kg . (m$^2$.s$^{-2}$ . m$^{-1}$) = kg.m.s$^{-2}$ = **N**

(the [–] sign specifies the spatial orientation of this action)

Example 1: calculate the reciprocal action that the tires exert on the globe when the a car of 1,500 kilograms (3,300 lb) describes a circular path of 100 meters radius (330 ft) at a speed of 20 meters per second (45 mph):

$$A = - 1,500 \times 20^2 / 100 = - 1,500 \times 400 / 100 = - 6,000 \text{ N}$$

The earth globe remains insensitive to this action because of the ratio of the masses: terrestrial globe (6 x 10$^{24}$ kg) *versus* car (1.5 x 10$^3$ kg) = 4 x 10$^{21}$.

Example 2: calculate the reciprocal action that a passenger weighing 100 kilograms exerts on the seat and the bodywork when the car describes a circular path of 100 meters radius (330 ft) at a speed of 20 meters per second (45 mph):

$$A = - 100 \times 20^2 / 100 = - 100 \times 400 / 100 = - 400 \text{ N}$$

The passenger feels perfectly this action which gives him the impression of weighing on the edge of the seat or the bodywork. The bodywork must be rigid enough and the seat must be strong enough to withstand this action.

## 4. Centrifugal force

$$F' = - M . Y$$

**F'**: centrifugal force, expressed in **N**
**M**: mass, expressed in **kg**
**Y**: transverse acceleration, expressed in **m.s$^{-2}$**

Consistency of the units: **F'** = kg . m.s$^{-2}$ = **N**

(the [–] sign specifies the spatial orientation of this force)

Example 1: calculate the force, called 'centrifugal force' that should be exerted on the center of mass of a stationary car with a mass of 1,500 kg to create, on the suspensions and the tires, the same effect as observed in the reality when the car is subjected to a transverse acceleration of 4 meters per square second:

$$F' = - 1,500 \times 4 = - 6,000 \text{ N}$$

Example 2: calculate the force, called 'centrifugal force', which should be exerted on the center of mass of a passenger weighing 100 kg sitting in a motionless car in order to create the same feeling as in the reality when the car is subjected to a transverse acceleration of 4 meters per square second:

$$F' = -100 \times 4 = -400 \text{ N}$$

Note 1: the sign [–] is required, it specifies that the spatial orientation of the centrifugal force conflicts the logic of the movement.

Note 2: this force is commonly referred to as '*centrifugal force*' which is an incorrect name since there is no curve, neither radius nor center (the car is motionless). The scientific name of this force is: imaginary force, fictional force, or pseudo-force.

Note 3: be careful not to confuse the centrifugal force with the reciprocal action: these two forces are equal, but the resemblance stops there:

- the reciprocal action is a real force that the passenger feels perfectly and exerts by contact with the seat or the bodywork, in response to the guiding force when the car describes a circular path.

- the centrifugal force is an imaginary force that is impossible to feel: it is the force that should be exerted on the center of mass of the passenger, if the car was motionless.

Note 4: the different calculations must be done in the order indicated. It is indeed impossible to directly calculate the centrifugal force without performing the intermediate calculations detailed above.

Note 5: Every scientific approach goes through four steps:

- form observation to *experiment* (here: a car which describes a circular path);

- from experiment to *measurements* (here: measuring the mass of the car, its speed and the radius of the curve);

- from measurements to *calculations* (here: calculation a guiding force and a transverse acceleration);

- from calculations to *reasoning* (here: the concept of centrifugal force).

This transition from concrete to abstract reasoning, from the real to the imaginary, has often been short-circuited, hence the confusion or misunderstanding about centrifugal force.

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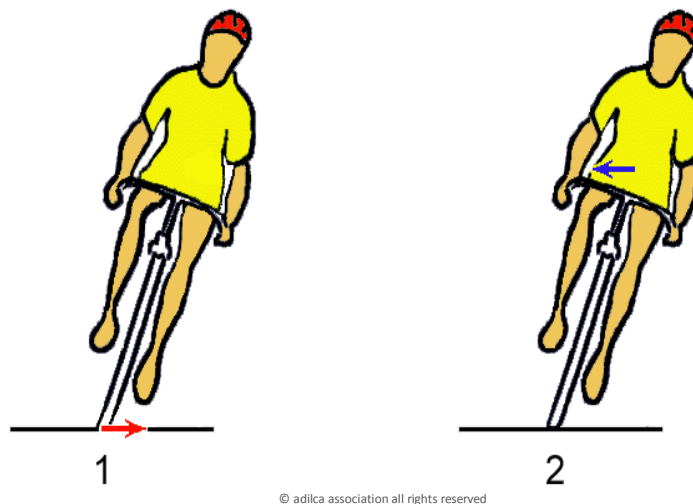
# IV.  CENTRIFUGAL FORCE: THE DRAWINGS

How to correctly draw an imaginary force? Let us take again the definition of the centrifugal force and apply it to the movement of a cyclist who describes a circular path:

'*Centrifugal force is the <u>imaginary force</u> that should be exerted on the center of mass of a <u>stationary cyclist</u> (we neglect the mass of the bicycle) to maintain it in equilibrium despite its inclination with respect to the vertical.*'

Let us recall the requirements of this definition:

1.  the cyclist is motionless;

2.  this force is imaginary, confirmed by the use of conditional: '*the force that should be exerted…*';

3. the impossibility of exerting a force directly on the center of mass of any mass.

That's why two drawings are needed:

Why two drawings? Only one would be enough to explain the movement of the cyclist, but if you want to draw the centrifugal force, you need a second one. Naturally, to clear up a misunderstanding, each drawing requires its own caption:

**Drawing 1:** this is the real description (said as '*dynamic*'): the cyclist is in motion, he describes a circular path, he has been deflected from a rectilinear direction by the *guiding force* exerted on the tires (red arrow).

**Drawing 2:** it is an imaginary description (said as '*static*'): the cyclist is motionless, he leans but does not fall, thanks to an imaginary force exerted on his center of mass (blue arrow). That is the *centrifugal force*.

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